Balanced Shaker and Sensor Placement for Modal Testing of Large Flexible Structures

by
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Abstract

Location of shakers and sensors for the dynamic testing and modal identification of the Z1 truss substructure of the International Space Station is discussed in this paper. It is assumed that every degree of freedom of the finite element model is a possible place for sensor location, but shakers can only be placed at selected locations (not all locations are technically suitable for the shaker placement). From this starting point, a subset of four shaker locations and a subset of about 400 sensor locations were selected, and subsequently used in the identification test of all the International Space Station (ISS) modes below 50 Hz of frequency. The locations were selected based on the shaker and sensor performance in the identification test that is comparable to the performance of the full sets. The performance is defined in terms of the Hankel singular values (HSV) that characterize the joint controllability and observability properties of each shaker and sensor for each natural vibration mode of the structure. The additive property of the structural Hankel singular values was used to locate the shakers and sensors. The placement results show good dynamic performance of the structure with the subset of shaker and sensor locations as compared to the structure with the full set of shakers and with sensors located at all degrees of freedom.

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1. Introduction

The location of sensors and shakers is an important problem in the dynamic tests of flexible structures, since test results depend substantially on these locations. For simple test articles, an experienced test engineer can determine appropriate sensor or shaker locations in an *ad hoc* manner. However, for first time testing of large and complex structures the placement of sensors and shakers is neither an obvious nor a simple task. In practice some heuristic means, combined with engineering judgments, are often used to determine shaker and sensor locations. In most cases shaker or sensor locations are varied during tests (using a trial and error approach) to obtain acceptable data to identify target modes.

The quality of a modal parameter identification process strongly depends on the quality of the measured response data, and the number of shakers/sensors on the structure at hand. Practical constraints dictate the use of a limited number of sensors and shakers during modal survey testing. Many authors have addressed sensor placement for structural modal parameter identification [1-4]. The generalized Hankel matrix, a function of the system controllability and observability grammians, is used by Lim [4] to develop an approach that determines sensor locations based on a given rank for the system observability matrix while satisfying modal test constraints. Kammer [3] identifies linear independence of target modes and the sensors that contribute the least are removed in an iterative process. Gawronski and Lim [2],[1] have developed the theory of the optimal sensor and shaker placement based on the balanced state space representation and shaker/sensor ranking using the Hankel singular values (HSVs). This approach is implemented and further developed in this paper.

A systematic analytical approach based on the controllability and observability properties of a linear time invariant structure is utilized herein to determine the optimal sensor and shaker locations. The joint controllability and observability properties are characterized by the system HSVs. Although sensor responses characterize the observability properties of a structure, nevertheless the structural controllability cannot be neglected: a well observed mode can be lightly excite, and therefore, its excitation amplitude could be below the instrument noise level.

HVS have additional physical interpretations, see Ref.[1], [5]: they are proportional to the modal response amplitude. Thus, the significance of shaker and sensor locations is related to the excitation level imparted to each mode, and consequently to the identifiability of the mode under a given shaker/sensor configuration. For this reason, HSVs construct a reliable shaker and sensor placement metric. Each structural mode is characterized by an almost equal pair of HSVs, which depends on the sensor and shaker location. This modal characterization of the HSVs can be achieved for flexible structures only, since their mode shapes remain almost invariant regardless of the location of sensors or shakers, and they are weakly coupled. Additionally, the HSVs for a set of sensors or shakers can be expressed as a root mean square (rms) sum of HSVs for each single sensor or shaker [1], [2]. Thus, the best shaker/sensor location is determined for each individual mode as the highest rms sum of HSVs of each shaker or sensor.

2. Structural Models

A flexible structure is a finite dimensional time-invariant, linear controllable and observable system with complex poles and small damping. A typical representation is the following second-order matrix differential equation.

$$M\ddot{q} + D\dot{q} + Kq = B_o u \qquad y = C_{oq}q + C_{ov}\dot{q} \tag{1}$$

where q is the $n ´m$ displacement vector u is the $s ´m$ input vector, y is the $r ´m$ output response vector, M is the $n ´n$ mass matrix, D is the $n ´n$ damping matrix, K is the $n ´n$ stiffness matrix, B_o is the $n ´s$ input matrix, C_{oq} is the $r ´n$ displacement output matrix, and C_{ov} is the $r ´n$ velocity output matrix, n represents the number of degree of freedoms of the system, r the number of outputs, and s the number of inputs. Its state-space representation is as follows:

$$\dot{x} = Ax + Bu\,, \quad y = Cx \tag{2a}$$

where

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M^{-1}B_o \end{bmatrix}, \quad C = \begin{bmatrix} C_{oq} & C_{ov} \end{bmatrix} \tag{2b}$$

and A is $2n ´2n$, B is $2n ´s$ *and* C is $r ´2n$.

In modal coordinates equations (2) take the form as in Ref.[1]

$$A_m = diag(A_{mi}), \quad B_m = \begin{bmatrix} B_{m1} \\ B_{m2} \\ \vdots \\ B_{mn} \end{bmatrix}, \quad C_m = \begin{bmatrix} C_{m1} & C_{m2} & \cdots & C_{mn} \end{bmatrix} \tag{3a}$$

for $i=1,\ldots,n$. where A_{mi}, B_{mi}, and C_{mi} are $2 ´2$, $2 ´s$, and $r ´2$ blocks, respectively.

$$A_{mi} = \begin{bmatrix} 0 & w_i \\ -w_i & -2z_i w_i \end{bmatrix}, \quad B_{mi} = \begin{bmatrix} 0 \\ b_i \end{bmatrix}, \quad C_{mi} = \begin{bmatrix} \dfrac{c_{qi}}{w_i} & c_{vi} \end{bmatrix} \tag{3b}$$

where $b_i = f_i^T B_o$, z_i is damping ratio, and w is natural frequency.

3. Flexible Structure Grammians

It is convenient to characterize the degree of observability and controllability of a structure by its corresponding grammians. The controllability grammian (W_c) and observability grammian (W_o) are derived as a solution of the following Lyapunov equations:

$$AW_c + W_c A^T + BB^T = 0, \tag{4a}$$

$$A^T W_o + W_o A + C^T C = 0 \tag{4b}$$

For stable A the solutions W_c and W_o are positive definite. The system is balanced if its grammians are equal and diagonal, as defined by Moore [6]

$$W_c = W_o = \Gamma, \quad \Gamma = diag(g_1, \ldots, g_{2n}), \quad g_i \geq 0, \quad i = 1, \ldots, n \tag{5}$$

where the positive variable g_i is the ith HSV of the system.

Two important properties of flexible structures turn out to be that their modal states are almost orthogonal and that their controllability and observability grammians are diagonally dominant, i.e.,

$$W_c \cong diag(w_{c1} I_2, \ldots, w_{cn} I_2), \qquad W_o \cong diag(w_{o1} I_2, \ldots, w_{on} I_2) \tag{6a}$$

with the diagonal blocks given by

$$w_{ci} = \frac{b_i b_i^T}{4 z_i w_i} \times I_2, \qquad w_{oi} = \frac{c_i^T c_i}{4 z_i w_i} \times I_2, \tag{6b}$$

and $c_i = \dfrac{c_{qi}}{w_i} + c_{vi}$, while I_2 is 2×2 identity matrix. Also, since the eigenvalues of the product are the HSVs then,

$$\Gamma^2 \cong W_c W_o,$$

where $\Gamma \cong diag(g_1, g_1, g_2, g_2, \ldots, g_n, g_n)$, and

$$g_i \cong \frac{\sqrt{(b_i b_i^T)(c_i^T c_i)}}{4 z_i w_i} \tag{7}$$

We note here that for flexible structures HSVs occur in pairs. Hence there will be 2*n* distinct HSV in general.

By re-scaling the modal representation one obtains

$$A_b \cong A, \qquad B_b \cong R^{-1}B, \qquad C_b \cong CR \tag{8}$$

where $R = diag(r_i I_2)$, $i=1,\ldots,n$, and $r_i = \left(w_{ci}/w_{oi}\right)^{1/4}$ are the scaling factors. This leads to a relation between the finite-element mode shape and the balanced (re-scaled) mode shape

$$f_{bi} = r_i f_i = \left(\frac{w_{ci}}{w_{oi}}\right)^{1/4} f_i \tag{9}$$

The above properties indicate that matrix A in the modal representation is almost the same as in the balanced representation, and is virtually independent of the sensor or shaker location. They also show that the orientation of the balanced and modal coordinates are nearly identical. However, the important difference lies in their scaling factors. The modal coordinates are non-unique because they depend on an arbitrary scaling factor of the natural mode, while the balanced modes are uniquely scaled to be equally controllable and observable.

4. Balanced Shaker and Sensor Placement

The sensor placement methodology presented here is virtually identical with the shaker location methodology, therefore only the first one is described here. The placement methodology is based on an approach given in Refs.[2] and [1], where the HSVs of the structural system are used to formulate metrics that quantify the degree of observability and controllability of a structure based on a given set of sensor locations in the modal coordinate system.

Consider the placement of p sensors from a set of P candidate locations. It has been shown in Refs.[2] and [1], that g_k, the HSV of the *k*th mode with the p sensors depends on $g_k(i)$, the HSVs of the *k*th mode of the *i*th sensor. That is

$$g_k \cong \sqrt{\sum_{i=1}^{p} g_k^2(i)}, \qquad k=1,\ldots,n \tag{10}$$

The significance of Eq. (10) is that it provides a means of evaluating the response of a given sensor for a particular balanced mode by looking at the individual values of the $g_k(i)$, where $g_k(i)$ is the *k*th HSV for the *i*th sensor location. Denote the scalar $g_k(P)$ as the maximum achievable controllability and observability corresponding to all candidate locations. This scalar is the rms sum of all the coefficients $g_k(i)$.

$$g_k(P) \cong \sqrt{\sum_{i=1}^{P} g_k^2(i)} \tag{11}$$

Let us define an index $s_k(i)$ of the ith sensor for the kth mode by:

$$s_k(i) = \frac{g_k(i)}{g(P)} \leq 1 \tag{12}$$

Then, the sum of all indices $s_k^2(i)$ over all of the p sensors for a given mode does not exceed 1:

$$s_k^2 = \sum_{k=1}^{p} s_k^2(i) \leq 1, \quad k = 1, \ldots n_2 \tag{13}$$

In this manner, the total contribution of the index for all sensors is decomposed into a sum of non-negative indices of each individual sensor.

5. Shaker and Sensor Placement Strategy for Large Structures

The key to the sensor placement strategy is the chosen metric of indices. In the proposed approach we choose those sensors with the largest indices. Note however, that in the case of large flexible structures there are local areas where a large number of sensors are located. Due to their close location the values of the placement index of a cluster of sensors are almost identical. Thus, a single sensor from this cluster would provide the same information as the whole local set. In order to solve this problem a method of elimination of similar sensors from a cluster is introduced via the correlation coefficient of the HSVs vectors as follows:

$$r_{ik}^2 = \frac{g_i^{2T} g_k^2}{\left\| g_i^2 \right\|_2 \left\| g_k^2 \right\|_2} \tag{14}$$

where g_i the vector of HSVs for the ith sensor. Denote e as a small positive number. Typically $e = 0.01 - 0.05$. If

$$r_{ik} \geq 1 - e, \tag{15}$$

then the two locations *i* and *k* are highly correlated, and one of them (the one with the smaller $s_k(i)$) is neglected.

For technical and economical reasons the number of sensors exceeds significantly the number of shakers. Thus the shaker selection comes first, as the less flexible procedure.

5.1 Shaker Placement Strategy

1. Place sensors at all accessible degrees of freedom.
2. Based on engineering experience, technical requirements, and physical constraints select the locations where shakers can be placed. In this way N_a candidate shaker locations have been selected.
3. For each mode (*k*), and each selected shaker location (*i*), determine the shaker placement index $s_k(i)$.
4. For each mode select the number n_{a1} of the most important shaker locations (those with largest $s_k(i)$). The resulting number of shakers n_{a2} for all the modes in consideration (i.e., $n_{a2} \leq n \times n_{a1}$) is much smaller than the number of candidate locations N_a, i.e., $n_{a2} << N_a$.
5. Check the correlation indices for the remaining n_{a2} shakers. Reject all but one shaker with correlation index higher than $1-e$. The resulting number of shakers is now $n_{a3} < n_{a2}$, typically $n_{a3} << n_{a2}$.
6. If the already small number n_{a3} is still too large, the shaker importance index, and modal importance index are re-calculated, and the shaker number is further reduced to the required one by reviewing the indices.

5.2 Sensor Placement Strategy

1. Shaker locations are already determined.
2. Select the areas where the sensors can be placed, obtaining N_s candidate sensor locations.
3. Determine the sensor placement indices $s_k(i)$ for the all candidate sensor locations ($i=1,\ldots,N_s$), and for all modes of interest ($k=1,\ldots,p$).
4. For each mode select n_{s1} the most important sensor locations. The resulting number of sensors n_{s2} for all the modes in consideration (i.e., $n_{s2} \leq n \times n_{s1}$) is much smaller than the number of candidate locations, i.e., $n_{s2} << N_s$
5. For the given small positive number e check the correlation indices for the remaining n_{s2} sensors. Reject the sensors with correlation indices higher than $1-e$. The resulting number of sensors is $n_{s3} < n_{s2}$, typically $n_{s3} << n_{s2}$.

6. Application to the International Space Station Z1 Truss Structure

The International Space Station truss structure Z1, see Fig. 1, is a large structure of a cubical shape with a basic truss frame and numerous appendages and attachments such as control moment gyros and a cable tray. The total mass of the structure is around 30,000 lb.

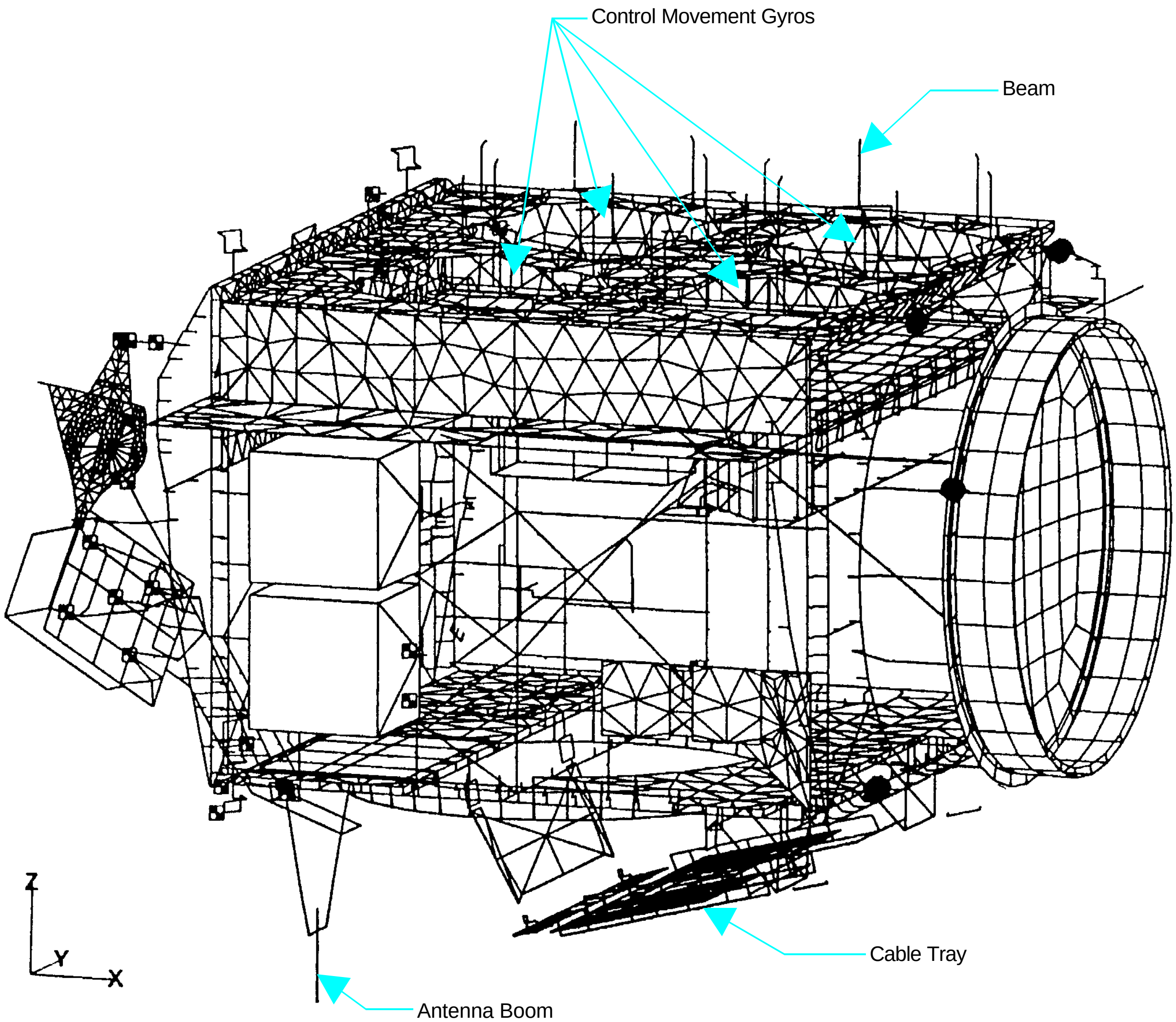

Fig. 1. Z1 Truss Structure

A complete finite element model of the structure was developed. It contains 11804 degrees of freedom with 56 modes below 70 Hz. The natural frequencies are listed in Table 1. The task is to identify all modes below 70 Hz by generating dynamic test data, with accelerometers used as sensors. This is a non-trivial undertaking that requires extensive pre-test analysis and careful planning of the shaker and sensor locations, especially if one does not have a freedom to repeat the test and modify the sensor/shaker location for re-testing.

Table 1. Z1 Truss Frequencies Below 70 Hz

9.318741E+00	3.852780E+01	5.484966E+01
1.611613E+01	3.969750E+01	5.752499E+01
1.978159E+01	4.011334E+01	5.776780E+01
2.354403E+01	4.100643E+01	5.919536E+01
2.562583E+01	4.180547E+01	5.974254E+01
2.619855E+01	4.235443E+01	6.036293E+01
2.761617E+01	4.254818E+01	6.159545E+01
2.940318E+01	4.268544E+01	6.339673E+01
2.975336E+01	4.399696E+01	6.412424E+01
3.012216E+01	4.481659E+01	6.512769E+01
3.040092E+01	4.635298E+01	6.524798E+01
3.122509E+01	4.870195E+01	6.607873E+01
3.200436E+01	4.921504E+01	6.695606E+01
3.269255E+01	4.979274E+01	6.722719E+01
3.306496E+01	5.026886E+01	6.737154E+01
3.410112E+01	5.058756E+01	6.763716E+01
3.498249E+01	5.089587E+01	6.784673E+01
3.520910E+01	5.175965E+01	6.879485E+01
3.625838E+01	5.284643E+01	6.928131E+01
3.643075E+01	5.364214E+01	
3.768060E+01	5.398895E+01	

6.1 Shaker Placement

The first part of the analysis involves selection of four shaker locations. The initial selection procedure is based on engineering judgment, practical experience, and physical constraints including the following criteria:

1. All target modes should be excited with relatively equal amplitudes.
2. Four shakers will be used such that the structure is excited in three axes.
3. Shakers are located such that each one provides structural responses that are neither identical nor close to each other.

The placement metric as defined in Eq. (12) evaluates the shaker locations using all candidate sensor outputs. Thus the shaker with highest index represents an appropriate input for the modal identification.

Table 2. Optimal Sensor Locations (degree of freedom)

45	90015-z	3377	93048-x	5704	95197-z	10438	282079-y	13457	14901
90	90030-z	3416	93061-x	5707	95198-z	10471	282090-y	13472	14973
405	90135-z	3488	93085-x	5710	95199-z	10472	282090-z	13482	14974
447	90149-z	3958	93241-z	5713	95200-z	10474	282091-y	13483	14976
450	90150-z	3961	93042-z	5716	95201-z	10475	282091-z	13832	14977
492	90166-y	3964	93043-z	5719	95202-z	10477	282092-y	13835	14979
495	90167-y	3993	94002-y	5722	95203-z	10483	282200-y	13838	14980
540	90182-y	3996	94003-y	5725	95204-z	10493	282203-z	13847	14982
1170	90394-y	3998	94004-x	5733	95207-y	10495	282204-y	13850	14983
1215	90409-y	4001	94005-x	5734	95208-y	10502	282206-z	13853	14985
1260	91013-y	4004	94006-x	5746	95211-z	10505	282207-z	13956	14986
1305	91028-y	4007	94007-x	5748	95212-y	10507	282208-y	13958	14989
1620	91133-y	4016	94010-x	5749	95212-z	10511	282209-z	13961	14992
1646	91142-x	4019	94011-x	6299	96010-x	10514	282210-z	13964	14995
1648	91142-z	4022	94012-x	6302	96011-x	10517	282211-z	13967	14997
1658	91146-x	4025	94013-x	6305	96012-x	10519	282250-y	13970	14998
1670	91150-x	4028	94014-x	6308	96013-x	10520	282250-z	13973	15000
1675	91151-z	4034	94016-x	6311	96014-x	10522	282251-y	13974	15001
1678	91152-z	4037	94017-x	6314	96015-x	10523	282251-z	13976	15004
1681	91153-z	4040	94018-x	6317	96016-x	10532	282254-z	14263	15010
1684	91154-z	4542	94185-y	6368	96033-x	10534	282255-y	14284	15371
1687	91155-z	4543	94185-z	6802	96177-z	10538	282256-z	14345	15374
1690	91156-z	4544	94186-x	6805	96178-z	10540	282257-y	14348	15377
1696	91158-z	4545	94186-y	6856	96195-z	10541	282257-z	14351	15386
1697	91159-x	4546	94186-z	6913	96214-z	10542	282258-x	14360	15389
1700	91160-x	4547	94187-x	6934	96221-z	10543	282258-y	14363	15492
1702	91160-z	4548	94187-y	6937	96222-z	10544	282258-z	14366	15487
1703	91161-x	4549	94187-z	6940	96223-z	10545	282259-x	14389	15490
1705	91161-z	4550	94188-x	6976	96235-z	10546	282259-y	14407	15492
1708	91162-z	4551	94188-y	6979	96236-z	10547	282259-z	14425	15493
1711	91163-z	4552	94188-z	6982	96237-z	10549	282260-y	14443	15495
1714	91164-z	4560	94191-y	6985	96248-z	10801	283007-y	14460	15496
1717	91165-z	4561	94191-z	6997	96242-z	10810	283010-y	14461	15499
1720	91166-z	4563	94192-y	7000	96243-z	10819	283013-y	14463	15502
1723	91167-z	4564	94192-z	7012	96247-z	10820	283013-z	14464	15505
1726	91168-z	4565	94193-x	7015	96248-z	10823	283014-z	14466	15508
1730	91170-x	4568	94194-x	7018	96249-z	10826	283015-z	14467	15511
1733	91171-x	4589	94201-x	7036	96255-z	10828	283016-y	14468	15513
1736	91172-x	4592	94202-x	7039	96256-z	10829	283016-z	14469	15514
1739	91173-x	4596	94203-y	7042	96257-z	10897	283039-y	14479	
2591	92101-x	4597	94203-z	7051	96260-z	10982	283089-z	14470	
3284	93017-x	4598	94204-x	7084	96271-z	11994	283093-z	14472	
3285	93017-y	4599	94204-y	7140	96290-y	12018		14473	
3287	93018-x	4601	94205-x	7143	96291-y	12021		14474	
3288	93018-y	4602	94205-y	7146	96292-y	12024		14476	
3290	93019-x	4603	94205-z	7149	96293-y	12048		14477	
3291	93019-y	4604	94206-x	7161	96297-y	12051		14479	
3300	93022-y	4605	94206-y	7164	96298-y	12078		14480	
3303	93023-y	5562	95150-y	7167	96299-y	12081		14482	
3352	93039-z	5563	95150-z	7170	96300-y	12084		14483	
3353	93040-x	5566	95151-z	7173	96301-y	12087		14484	
3355	93040-z	5568	95152-y	7176	96302-y	12090		14485	
3356	93041-x	5569	95152-z	7182	96304-y	12093		14486	
3358	93041-z	5584	95157-z	7188	96306-y	12096		14487	
3359	93042-x	5587	95158-z	7200	96310-y	12174		14488	
3361	93042-z	5590	95159-z	7206	96312-y	12177		14489	
3362	93043-x	5593	95160-z	7209	96313-y	12180		14496	
3364	93043-z	5599	95162-z	7299	97002-y	12248		14845	
3365	93044-x	5605	95164-z	7314	97007-y	12416		14859	
3367	93044-z	5626	95171-z	7493	97067-x	12682		14865	
3368	93045-x	5629	95172-z	8670	98144-y	12685		14865	
3370	93045-z	5660	95183-x	8722	98161-z	12688		14874	
3371	93046-x			9921	281078-x	12691		14874	
3373	93046-z			10429	282076-y	12694		14875	
3374	93047-x			10431	282077-x	13451		14889	
3376	93047-z			10432	282077-z			14892	

The procedure for the shaker placement was carried out in three steps:

1. The first step involves evaluation of the HSVs of the shakers with the sensors distributed all over the structure, and determining the importance indices of each shaker. The shakers with the highest index are kept. The process requires a predetermined threshold of acceptability for a shaker.
2. The second step entailed the analysis of correlation between the HSVs for a given shaker and all the remaining shakers. All highly correlated shakers are deleted from the list, and only a small number of shakers that produce significantly different and uncorrelated HSVs is kept. The threshold for the acceptance or rejection of a shaker is based on the judgment of the test engineer. This sorting procedure results in a number of shakers that is usually larger than the target number, which leads to the third step.
3. From the remaining small number of shakers select the target number of shakers, that are spatially well separated and have the highest indices. This final step will result in a set of shaker locations that practically and technically satisfy the requirements.

The Z1 Truss structure drawings and the finite element model were examined and the shaker candidate locations at 2256 out of the 11804 translational degree of freedom were selected. The selection was based on accessibility of the locations, strength of the structural parts, mass of components or sub-components, and local flexibility. It was assumed that the movement of all degree of freedom was sensed using accelerometers as measurement devices. In this analysis the HSVs for each shaker were determined (the physical meaning of the HSV is the response amplitude picked up by each of the sensor under equal excitation forces by each of the 2256 shakers), and used to evaluate the shaker importance indices. For each of 56 modes the 6 most important shakers were selected, obtaining 268 shaker locations (it is less than 336, because some locations were the same for two or more modes). Next, the correlation coefficients of the HSVs for each shaker location were obtained. Those highly correlated were discarded and the one with the highest index of all of the well correlated shakers was kept.

This process reduced the number of shakers down to 52 locations. The next step of the process involved are-evaluation of the importance indices of each shaker and their comparison to the threshold value. This step reduces the number of shaker locations down to 7. The final step involved evaluation of the actual location of these shakers using the finite element model simulations, along with determination of accessibility, structural strength and the importance index. The final four shakers were located at the nodal points shown in Fig. 1, black spots. These four locations are essentially near the four corners of the truss and according to test experts they are reasonable locations.

6.2 Sensor Placement

The sensor selection criteria includes:

1. Establishing the maximum allowable number of sensors. In our case is was 400.
2. Determination of the sensor indices; for each mode the sensors with the highest indices were selected.
3. Taking into account that local modes with a limited number of participating degrees of freedom require significantly less sensors than the global modes (the latter ones need the measurement points distributed all over the structure).
4. Based on the above criteria and using the performance index of Eq. (12), followed by the correlation procedure, a set of sensors was selected which should provide test data appropriate for the identification of the target modes.

The excitation level of each mode with the set of four shakers is represented by the HSVs and are shown in Fig. 2a. It can be seen that some modes are weakly excited, providing weaker measurement signals, and are more difficult to identify. Moreover, the amplitude of the shaker inputs determines the relative output response. Figure 2b presents an overview of the sensor importance index for each sensor as the sum total of all the indices for all modes. It is obvious from this figure that the degrees of freedom that have a greater amplitude of modal vibrations or a stronger participation in the dynamics of the structure have higher indices. By looking at the sensor importance indices for a particular mode one can roughly evaluate the participation of each mode at a particular sensor location, or at a particular degree-of freedom. The highly participating modes have a high index at this location. A set of pictures presented in Fig. 3 shows the placement indices of each sensor. The plots show the indices for the first 10 modes. The first one (Fig. 3a) one is a global (or a system) mode with indices for all sensors almost the same. The second (Fig. 3b) is a global mode of more complex configuration. The third, fourth, fifth, and seventh modes (Figs. 3c, d, e, g) show more dominant responses from the cable tray attachment. The sixth mode is more global, with domination of the local motion at locations 1000-2000, the attachments and cross-beams near the circular dish on a side of the Z1. The eighth and ninth modes (Figs. 3h, i) are local modes of the control moment gyros - see the four columns sticking up at the end. The last one (Fig. 3j) shows a highly dominant mode of a beam sticking out of Z1.

Figure 4 indicates the selected sensor locations. It can be observed that most of the sensors are located in and around the control moment gyros (see Fig. 4) and the cable tray (see Fig. 1) since 13 out of the 56 modes involve extensive control moment gyro movement and 9 are mostly cable tray modes. Most of the 56 modes are local modes that require concentrations of sensors at the particular locations seen in Figure 4.

7. Conclusions

A methodology for placing shakers and sensors for modal testing of a large flexible structure has been presented. The initial ranking of shakers and sensors was based on their HSVs for each mode. The further elimination is based on the correlation of the HSVs of the remaining shakers or sensors. The approach is illustrated with the ISS module, by the analytical determination of the instrumentation locations, that maximize the controllability and the observability of the test article.

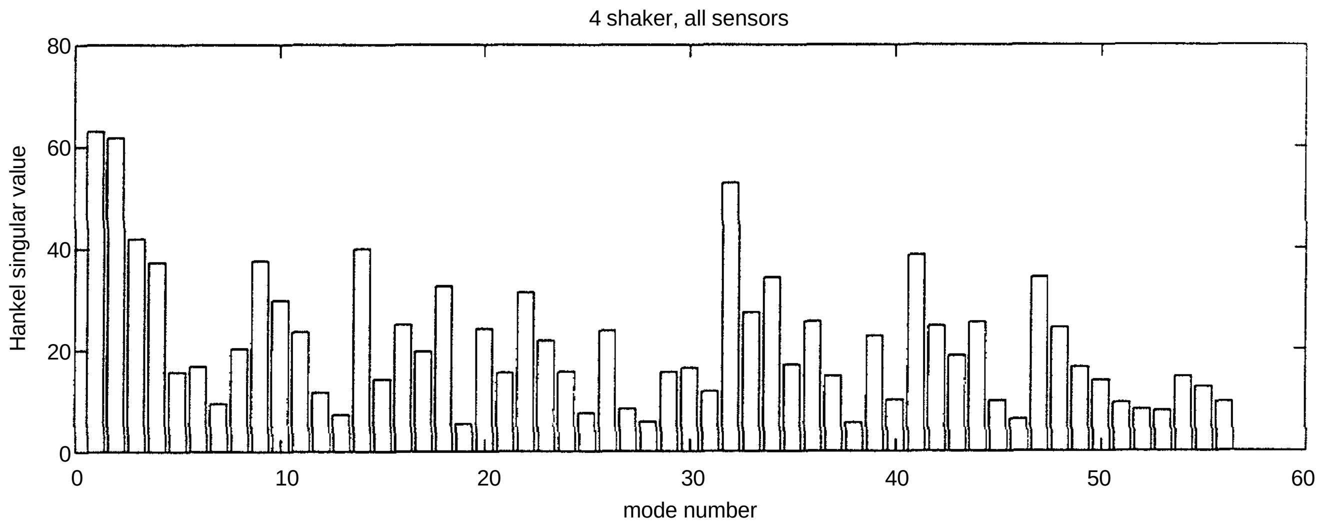

Fig. 2a. Cumulative Model Response for All 6 Modes

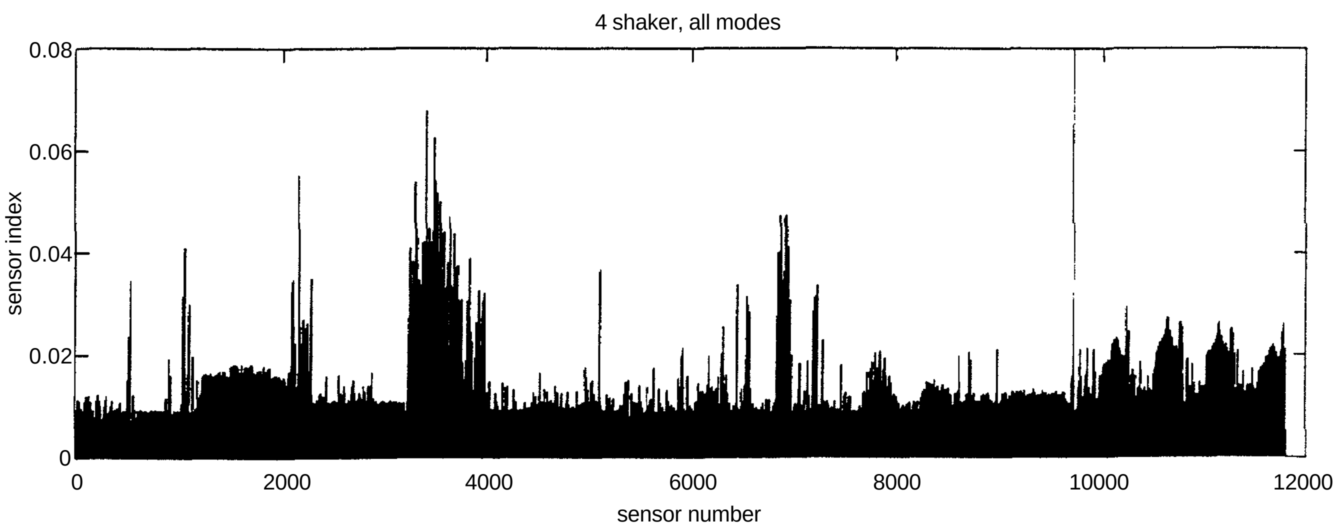

Fig. 2b. Total Response by Each Sensor for All Modes

Fig. 2. Response by Sensor

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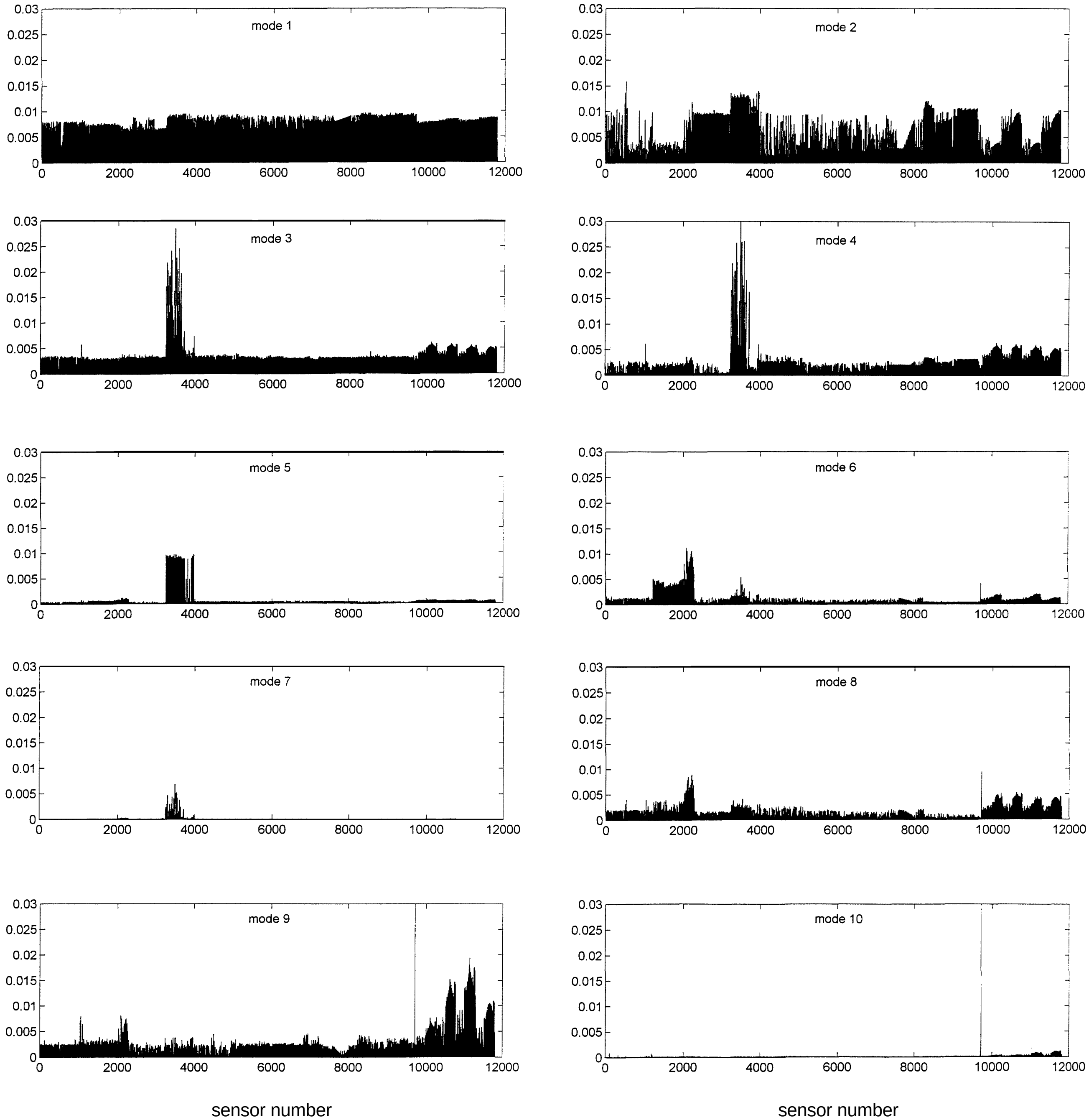

Figure 3. Placement Indices of Each Sensor

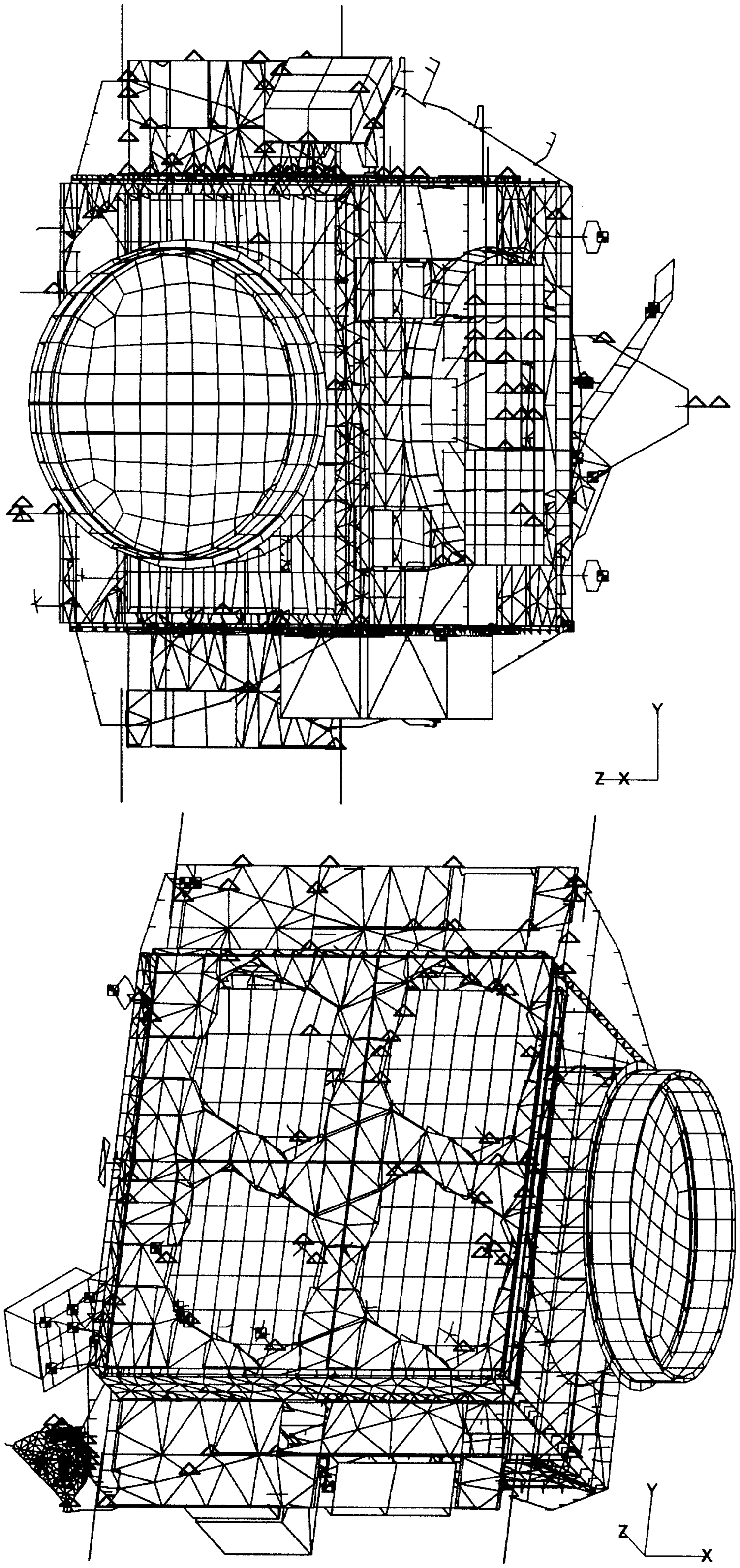

Fig. 4. Final Configuration for the Z1.